MA 234, Spring 2016

Assignment 8

This homework is due Friday April 1.

There are total 55 points in this assignment. 49 points is considered 100%. If you go over 49 points, you will get over 100% for this homework (but not over 115%) and it will count towards your course grade.

Collaboration is welcome. If you do collaborate, make sure to write/type your own paper *and give credit to your collaborators in your pledge.* Your solutions should exhibit your work and contain full proofs. Bare answers will not earn you much.

This assignment covers Sections 5.4, 5.5, and 6.1 of Textbook.

1. Trig functions

(1) [10pt] Use expression of trigonometric and hyperbolic functions through the exponential function to establish the following:
   (a) $\sin\left(\frac{\pi}{2} - z\right) = \cos z$.
   (b) $\tanh(z + i\pi) = \tanh z$.
   (c) $\cos 2z = \cos^2 z - \sin^2 z$.
   (d) Find a similar formula for $\cosh 2z$.

(2) [5pt] Show that $\sin \overline{z} = \overline{\sin z}$ and that $\sin \overline{z}$ is not analytic at any point.

(3) [10pt] Find *all* values of the following. (Express them as $x + iy$.)
   (a) $\arcsin \frac{\sqrt{3}}{2}$.
   (b) $\arcsin 3$.
   (c) $\arccos 3i$.
   (d) $\arctan i$.
   (e) $\arctan 2i$.

(4) [10pt] Show that
   (a) $\arccos z = -i \log\left(z + i(1 - z^2)^{\frac{1}{2}}\right)$.
   (b) $\operatorname{arcsinh} z = \log\left(z + (1 + z^2)^{\frac{1}{2}}\right)$.
   (c) $\frac{d}{dz} \arccos z = \frac{-1}{(1-z^2)^{\frac{1}{2}}}$.

2. Complex integral

(5) [10pt] Find the following integrals (either by expressing them through real and imaginary part, or by guessing the complex antiderivative):

   (a) $\int_0^1 (3t + i)^2 dt$,
   (b) $\int_0^{\frac{\pi}{2}} \cosh(it) dt$,
   (c) $\int_0^2 \frac{t}{t+i} dt$,
   (d) $\int_0^1 e^{it+2t} dt$.

(6) [5pt] Let m, n be integers. Show that

$$\int_0^{2\pi} e^{imt} e^{-int} dt = \begin{cases} 0 \text{ when } m \neq n, \\ 2\pi \text{ when } m = n. \end{cases}$$

(7) [5pt] Show that $\int_0^{\infty} e^{-zt} dt = \frac{1}{z}$ provided $\operatorname{Re}(z) > 0$. Why is the latter condition important?